Universidade Vigo

Contributions in Cooperative Game Theory and Applications

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The main purpose of this dissertation is to tackle several topics in cooperative game theory and its applications. In this sense, this manuscript is composed by three independent parts, each of one devoted to the analysis of different situations.

Part I. On cooperative inventory models. A multi-agent inventory problem is a situation in which several agents face individual inventory problems and agree to coordinate their orders with the objective of reducing their costs. This part of the manuscript is composed of two independent chapters, each of them dealing with a new multi-agent inventory problem.

Chapter 1. A multi-agent inventory problem with two acquisition costs. Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2015) analyse a multi-agent inventory problem in which several farmers cooperate to make joint orders of animal feed, the cost of a usual feeding ration being equal to the cost of a shortage feeding ration. However, this equality assumption is not acceptable in some cases. Thus, along this chapter, we analyse the corresponding multi-agent inventory problem with two acquisition costs.

This chapter is mainly based on Saavedra-Nieves, García-Jurado, and Fiestras-Janeiro (2018b).

Chapter 2. A multi-agent inventory problem with general transportation costs. Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2014) study a multi-agent inventory system where each agent has a deterministic demand and a capacitated warehouse with constant holding costs. Besides, shortages are not allowed and lead-time is constant. A key assumption is that agents are located in a line route in such way that the transportation costs depend on the maximum distance to the supplier. In this chapter, we take the case in which the variable cost is given by a general function. Under these assumptions, each agent faces a continuous-review inventory problem, with a deterministic and linear demand, without holding costs, with a limited capacity warehouse and with general transportation costs. We analyse the case of the cooperation of the agents when they agree to place joint orders.

This chapter is mainly based on Saavedra-Nieves, García-Jurado, and Fiestras-Janeiro (2018c).

Part II. On sequencing situations with non-linear costs. Sequencing problems describe those situations where a set of jobs has to be processed in a collection of available machines. In this class of problems, an initial order for processing the jobs is assumed and each of them has associated a specific cost function which generally depends on the time in the machine. This second part is composed of a single-chapter, which describes sequencing situations with exponential and logarithmic costs.

Chapter 3. A sequencing problem with exponential and logarithmic costs. Curiel, Pederzoli, and Tijs (1989) firstly analyse from a game-theoretic approach the standard case of Smith (1956), where the costs of processing the jobs are given by linear functions of the time in the machine. Borm, Fiestras-Janeiro, Hamers, Sánchez, and Voorneveld (2002) additionally provide a condition for the convexity of the saving games associated with these situations. The aim of this chapter is to analyse sequencing problems with exponential and logarithmic costs by processing the jobs in a single-machine. Under these new considerations, we obtain results about the optimal order and we analyse the savings obtained by rearranging the jobs using cooperative game theory.

This chapter is mainly based on Saavedra-Nieves, Schouten, and Borm (2018).

Part III. On sampling methods and coalitional values. Cooperative game theory focuses on the definition of allocation procedures of the costs/profits by the collaboration of the players. Among others, coalitional values take relevance in this setting. However, their exact determination becomes a difficult task when the number of involved players enlarges. Due to the interpretation of some coalitional values as the average of the marginal contributions, approximation methods based on sampling techniques may reduce considerably these efforts. The third part of the manuscript is organized in two chapters, each of them, including new sampling proposals for estimating coalitional values.

Chapter 4. Estimation of the Owen value based on sampling. Fernández-García and Puerto-Albandoz (2006) and Castro, Gómez, and Tejada (2009) provide a procedure based on simple random sampling with replacement to estimate the Shapley value (Shapley, 1953). Along this chapter, we introduce a variation of the above-mentioned procedure to approximate the Owen value of general TU-games (Owen, 1977). We analyse our proposal from a statistical perspective and we check how it performs on some examples extracted from the literature.

This chapter is mainly based on Saavedra-Nieves, García-Jurado, and Fiestras-Janeiro (2018a).

Chapter 5. Estimation of the Banzhaf-Owen value based on sampling. Bachrach et al. (2010) also provide an estimating procedure to approach the Banzhaf value (Banzhaf, 1965) of simple games. In this chapter, we propose some variations of Bachrach et al.'s procedure to estimate the Banzhaf-Owen value of general TU-games (Owen, 1982). First, we propose an approximation procedure based on simple random sampling without replacement. Secondly, we introduce an alternative estimation method to approximate the Banzhaf-Owen value, based on a two-stage sampling procedure. Both methodologies are theoretically analysed and their performance on several examples in the literature is also evaluated, with positive results.

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